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## NUMERICAL MODEL OF EXTERNAL HEAT EXCHANGE IN A GAS-ELECTRIC GLASSMAKING FURNACE

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A concept for a mathematical model of a gas-electric glassmaking furnace is formulated. A system of equations for calculating external heat transfer is presented. The computational boundary conditions are examined.

***Key words:*** glassmaking furnace, mathematical model, boundary conditions.

Glass is made in regenerative furnaces with transverse and horseshoe motion of the fuel-combustion products in the melting chamber. The furnaces are divided into gas and gas-electric according to the method of heating. Glassmaking furnaces can also be classified according to their capacity and the chemical composition of the glass. It is obvious that the heat and mass transfer processes in each particular furnace develop according to their own definite and characteristic way, which is manifested in the quantitative operational parameters of a furnace. On a qualitative level the flow of thermophysical processes in furnaces is of a general character. For this reason, to construct a mathematical model of a gas-electric furnace one must start from the basic principles of the general theory of furnaces, where the energetics of the technological process is the principal and necessary condition for process flow.

The energetic essence of furnace operation unified by the concept of the thermal operation of furnaces is the most important part of the science of the furnace design and calculation. The energetic essence of the operation of the entire diversity of furnaces by type, dimensions, and purpose can be described by a limited number of physical models, by studying which the fundamental principles of furnace design can be determined.

The physical model of a glassmaking furnace can be represented as two zones (Fig. 1): the technological process zone (TPZ — the melting tank) and the heat generation zone (HGZ — the melting chamber). These zones are divided by the molten glass surface $F$ and are separated from the surrounding medium by the refractory masonry $M$. Since a glassmaking furnace is a piece of technological equipment, the TPZ is considered to be the primary zone and the HGZ the secondary zone, whose function is to create definite energy conditions in the technological process zone.

In gas glassmaking furnaces (Fig. 1*a*), heat appears in the melting tank as a result of heat transfer from the melting chamber. The processes ensuring heat flow from the HGZ into the TPZ are conventionally called determining and the furnaces are said to be heat-exchanger furnaces. The intensity of the technological process in furnaces of this type will depend on the efficiency of external heat exchange and the

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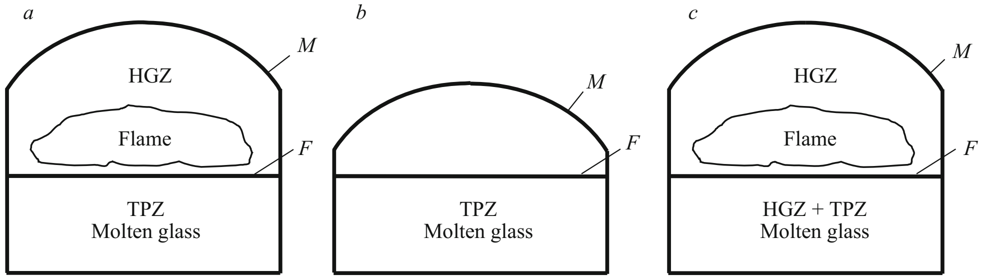


**Fig. 1.** Diagram of a physical model of gas (*a*), electric (*b*), and gas-electric (*c*) glassmaking furnaces.

utilization of the heat energy transferred into the melting tank. The processes on which the distribution and utilization of heat in the technological process zone depend are determined.

The determining and determined processes can be of different character. It depends on the conditions of heat flow into the tank. For example, in electric glassmaking furnaces heat appears directly in the technological process zone (Fig. 1*b*). For this reason, furnaces with this type of heating regime are called heat-generator furnaces.

A mixed heat-generation regime, where the main source of heat delivery to the furnace is the chemical energy of hydrocarbon fuel (natural gas, liquid fuel), is realized in gas-electric glassmaking furnaces (Fig. 1*c*). The heat introduced directly into the molten glass by means of electric power (5 – 20% of the heat load of a furnace) is considered to be an additional source of energy. It is conventionally called an auxiliary electric heater (AEH).

Thus, the conceptual model of the thermal operation of glassmaking furnaces can be represented as a collection of heat generation processes and external heat-exchange in the melting chamber as well as the hydrodynamics and internal heat-exchange in the melting tank. The coupling of the external and internal heat and mass transfer provides for the physical integrity of the thermal operation of a furnace.

A unique feature of glassmaking furnaces is that the surface of the molten glass can be used to couple the determining and determined thermal processes. The choice is due to the fact that melting of the batch and production of the primary melt occur on the surface of the tank, most of the heat is transferred through this surface from the melting chamber into the technological process zone, and finally the temperature distribution on the surface of the molten glass is a principal boundary condition for the hydrodynamics of the melting tank. In other words, the effect of the determining processes on glassmaking is the formation of the temperature field of the surface of the molten glass. Such an approach, which does not distort the physical essence of the thermal operation of the furnace, greatly simplifies the mathematical formalization of the thermophysical processes involved in glassmaking, which can be represented by a system of mathematical models of external and internal heat and mass transfer, which are coupled by differential specification of the boundary conditions on the molten-glass surface.

For the external problem these are boundary conditions of the second kind. In this case numerical modeling of the mechanisms of external heat exchange with different flame organization and variable furnace capacity becomes possible. The numerically determined temperature field of the melt surface objectively reflects the intercoupled character of the heat exchange between the heat generation and technological process zones. This gives a basis for using the computed tank surface temperatures as boundary conditions of the first kind when modeling the internal problem. The correctness of such an approach to prescribing a very important condition for modeling the hydrodynamics and heat exchange in the melting tank is due to the thermophysical nature of the formation of the convective motion of the molten glass in the melting tank of a glassmaking furnace.

Boundary conditions of the first kind remain admissible for the adiabatic surfaces of the melting chamber — the inlets of the air channels of the burners and the sections of the side walls through which the batch is loaded into the furnace. At the same time boundary conditions of the third kind, realized by setting the average heat-transfer coefficient and the temperature of the surrounding medium, must be adopted on the furnace-barrier surfaces. Such an approach makes it possible to take account of the design features of the masonry of individual sections of the melting part of the furnace as well as the effect of the furnace capacity and flame organization on the energy parameters of external heat exchange.

The concept proposed for a mathematical model predetermines the functional purpose of the models of external and internal heat and mass transfer as well as the requirement for the numerical schemes used to solve them. The mathematical equations of the external problem and their discrete analogs are intended for formalization and calculation of the energy aspects of glassmaking. The final equations of the mathematical model of the thermal operation of a furnace — the equations of hydrodynamics and internal heat exchange — are self-consistent. For their solution, the external problem need only provide information on the surface temperature field of the molten glass. Although these equations are a formalization of convective heat-exchange between the melt and melting-tank surfaces bounding its motion, i.e., they also describe the laws of heat transfer, the purpose of this block of the mathematical model is purely technological. It is intended for determining the conditions for a rational distribution of heat in the tank that are required to secure the needed furnace capacity and molten-glass quality.

When using the resolvent zonal method to calculate heat exchange in a three-dimensional melting chamber of a furnace the differentiated heat flows along the melting tank, which are due to endothermal glassmaking processes and heat losses through the tank masonry, are taken into account in the source term of the nonlinear equations of heat transfer and heat balance for the surface zones of the molten glass.

A strict mathematical description of the thermal operation of a glassmaking furnace using an AEH requires that the electric field in the melt be calculated accurately. At the present time it is impossible to perform such a calculation without having sufficiently reliable data on the electromagnetic properties of the molten glass and methods for calculating the parameters of the electric field and the electric discharge between several electrodes. Taking account of only the integral characteristics of such a discharge does not solve the problem, because the equations of electrodynamics do not fit well with the resolvent zonal method of calculating radiative-convective heat exchange. One possible way out this situation could be to include the AEH power in the source terms of the equations for the surface zones of the molten glass.

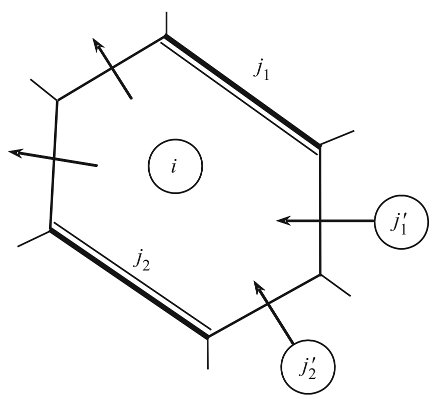


**Fig. 2.** Zones adjoining the *i*th volume zone.

To construct a numerical model of external heat exchange we assume that the gas motion is stationary, while the distribution of the mass velocity in the gas volume is described by a known function $\rho v(N)$, where $N$ is a point in the gas volume. Then, the mass flux $G$, kg/sec, can be determined for any section of the surface separating the volume zones:

$$G = (\rho v)_n F,$$

where $(\rho v)_n$ is the average value of the projection of the mass velocity of the gas on the normal to the surface $F$.

The average value of the heat emission coefficient $\alpha$, W/(m$^2 \cdot$ K), calculated using the generalized relation in [1] is prescribed for the surfaces zones.

Let us examine the zonal expression for the convective heat flux. Let $m$ volume and $n$ surface zones with the total number of zones being $l = m + n$ be separated in the system under study. For each $i$th zone the resulting heat flow is comprised of the resulting radiation flow $Q_i^{\text{rad}}$, kW, and the convective heat flux $Q_i^{\text{con}}$, kW. The convective heat flux of the surface zones is understood to be the convective heat emission flow according to Newton's law. For the volume zones this is the sum of the heat transferred by the moving medium and the convective heat-emission flow to adjoining surface zones.

The total (resulting) heat flow $\widetilde{Q}_i$ is

$$\widetilde{Q}_i = Q_i^{\text{rad}} + Q_i^{\text{con}} .$$

In the present case, in contrast to the problem of radiative heat exchange, it is the quantity $\widetilde{Q}_i$ that must be determined for type-I zones and must be prescribed as condition for type-II zones. For example, in the absence of heat release in some volume zone it is necessary to set $\widetilde{Q}_i = 0$ or $Q_i^{\text{rad}} + Q_i^{\text{con}} = 0$ in accordance with the fact that the heat emitted by this zone in a stationary regime must be compensated by convective delivery of heat from neighboring zones. For zones on the surface of the masonry the quantity $\widetilde{Q}_i$ is the flow of heat losses $Q_i^{\text{loss}}$. For ideal thermal insulation of the masonry $\widetilde{Q}_i = 0$ and $Q_i^{\text{rad}} + Q_i^{\text{con}} = 0$, i.e., the heat flow delivered to the solid surface by convection must be returned into the system as a result of heat exchange by radiation.

Thus, the state of the system with complex heat exchange is characterized by the zonal values of the temperatures $T_i$ and the resulting heat flows $\widetilde{Q}_i$. To construct a closed system of equations the relations describing radiative heat exchange must be supplemented by the dependence of the convective heat flow for each $i$th zone on its temperature and the temperatures of the adjoining zones.

For the prescribed values of the heat emission coefficients $Q_i^{\text{con}}$ for surface zones is determined from the equation

$$Q_i^{\text{con}} = \alpha_i (T_j - T_i) F_i \quad \text{for } i = 1, ..., n, \tag{1}$$

where $T$, K, is the average temperature of the zone and $j$ is the number of the volume zone bounding the $i$th surface zone.

For volume zones the desired relation will have a more complicated form, since it must take account of the heat emission of the products of combustion to the surface zones and the convective transfer of heat between the volume zones.

Let us examine the $i$th volume zone ($i = n + 1, ..., l$). We shall denote the numbers of the surface and adjoining volume zones as follows (Fig. 2): $j_1, j_2, ...$ — surface zones; $j'_1$, $j'_2$, ... — volume zones, of which the products of combustion enter the zone under consideration.

We shall denote by $G_{j'i}$ the mass flow transferred to the $i$th zone from the $j'$ zone and we shall take account of the fact that an additional mass flow can enter the $i$th volume zone through the adjoining surface zones.

For a glassmaking furnace the additional mass flow for the first volume zone in the path of the gases includes the fuel mass flow $G_i^{\text{g}}$, the air mass flow $G_i^{\text{a}}$, and the mass flow $G_i^{\text{p.c}}$ of the products of combustion drawn to the root of the flame in the circulation loops. Then the total mass flow entering (and leaving) the $i$th volume zone will equal

$$\sum_{j'} G_{j'i} = G_i^{\text{g}} + G_i^{\text{a}} + G_i^{\text{p.c}}.$$

Using the notation introduced above, we shall now express the individual components of the convective heat flow for the $i$th volume zone:

– heat flow emitted by the gas to the surface zones

$$\sum_j \alpha_j (T_j - T_i) F_j ;$$

– heat flow obtained by the $i$th zone due to the motion of the gas

$$\sum_{j'} c_{j'} (T_{j'} - 273) G_{j'i} + c_{\text{g}} (T_{\text{g}} - 273)\, G_i^{\text{g}} + c_{\text{a}} (T_{\text{a}} - 273)\, G_i^{\text{a}} + c_{\text{p.c}} (T_{\text{p.c}} - 273)\, G_i^{\text{p.c}};$$

– heat flow lost by the $i$th zone due to the motion of the gas

$$-c_i(T_i-273)\left(\sum_{j'} G_{j'i}+G_i^{\mathrm{g}}+G_i^{\mathrm{a}}+G_i^{\mathrm{p.c}}\right),$$

where $c$ is the specific heat capacity of the medium, kJ/(kg · K), and $T_{\mathrm{p.c}}$ is the temperature of the products of combustion entering from outside the system under study, K.

Summing these three components we obtain the complete expression for the convective heat flow to the $i$th volume zone, which can be written in a compact form as a linear combination of temperatures:

$$Q_i^{\mathrm{con}}=\sum_k g_{ik}T_k+g_i^0, \tag{2}$$

where $g_i^0$ is the sum of the terms which do not depend on the zonal temperatures and $g_{ki}$ is the coefficient of convective exchange, determining the contribution of the $k$th zone to the convective flux of the $i$th zone.

For the volume zones ($i = n + 1, ..., l$) the coefficients $g_{ki}$ are determined by the following expressions:

$$\begin{aligned}g_i^0 &= c_{\mathrm{g}}T_{\mathrm{g}}G_i^{\mathrm{g}}+c_{\mathrm{a}}T_{\mathrm{a}}G_i^{\mathrm{a}}+c_{\mathrm{p.c}}T_{\mathrm{p.c}}G_i^{\mathrm{p.c}}+\\ &+273\left[\sum_{j'}(c_i-c_{j'})G_{j'i}+(c_i-c_{\mathrm{g}})G_i^{\mathrm{g}}+\right.\\ &\left.(c_i-c_{\mathrm{a}})G_i^{\mathrm{a}}+(c_i-c_{\mathrm{p.c}})G_i^{\mathrm{p.c}}\right];\end{aligned}$$

$$g_{ki}=\begin{cases}0 & \text{for}\quad k\neq j, k\neq j', k\neq i;\\ \alpha_j F_j & \text{for}\quad k=j;\\ c_{j'}G_{j'i} & \text{for}\quad k=j';\\ -\left[\sum_j \alpha_j F_j+c_i\left(\sum_{j'}G_{j'i}+G_i^{\mathrm{g}}+G_i^{\mathrm{a}}+G_i^{\mathrm{p.c}}\right)\right] & \text{for}\ k=i,\end{cases}$$

where $j$ is the number of the adjoining surface zones and $j'$ are the numbers of volume exchange zones from which gas enters the $i$th zone.

The expression (1) for the convection heat-emission flow $Q_i^{\mathrm{con}}$ for the surface zones ($i = 1, ..., n$) can be represented in form of Eq. (2). Then

$$g_i^0=0;$$

$$g_{ki}=\begin{cases}0 & \text{for}\quad k\neq j, k\neq i;\\ \alpha_i F_i & \text{for}\quad k=j;\\ -\alpha_i F_i & \text{for}\quad k=i,\end{cases}$$

where $j$ is the number of the adjoining volume zone.

Using the general form of the equations of the resolvent zonal method for radiation problems and replacing in them the resulting radiation flows $Q_i^{\mathrm{rad}}$ by the differences $\widetilde{Q}_i-Q_i^{\mathrm{con}}$ for $i_1 = l_1, ..., l$ we obtain:

for type-I zones

$$\widetilde{Q}_i=\sum_k(A_{ki}T_k^4+g_{ki}T_k)+g_i^0\quad \text{for}\ i=1,...,l_1; \tag{3}$$

for type-II zones

$$\sum_k(A_{ki}T_k^4+g_{ki}T_k)+g_i^0-\widetilde{Q}_i=0\quad \text{for}\ i=l_1,...,l, \tag{4}$$

where $A_{ki}$ is the coefficient of radiative exchange between the $k$th and $i$th zones; $l = l_1 + l_2$ is the total number of zones; and, $l_1$ and $l_2$ are the numbers of type-I and -II zones.

To solve the problem the temperatures of type-II zones must be determined from the system of equations (4) and the values found substituted into the relations (3). In order to choose a method for solving the system of equations (4) we rewrite it, separating on the left-hand sides of the equations the terms of the sum which contain the unknown temperatures of the type-II zones. For this we denote the sum of the first $l_1$ terms in the expressions $\sum_k(A_{ki}T_k^4+g_{ki}T_k)$ in terms of the parameters $z_i$:

$$z_i=\sum_{k=1}^{l_1}(A_{ki}T_k^4+g_{ki}T_k).$$

The quantities $z_i$, characterizing for each $i$th zone the part of the resulting heat flow that is due to the interaction of this zone with the type-I zones, are determined by the prescribed temperatures of these zones and can be calculated beforehand. Substituting these quantities into Eq. (4) we obtain a system of equations in which the terms containing the desired temperatures of the type-II zones:

$$\sum_{k=l_1+1}^{l}(A_{ki}T_k^4+g_{ki}T_k)+z_i+g_i^0-\widetilde{Q}_i=0$$
$$\text{for}\ i=l_1+1,...,l. \tag{5}$$

In summary, a feature of the description of the complicated heat exchange in the melting chamber of a glassmaking furnace is that the system of zonal equations (5) is nonlinear with respect to the temperatures of type-II zones. Compared with the classical zonal method, the system (5) of nonlinear equations obtained above is much easier and more convenient to solve numerically. In the first place, the number of equations in the system reduces to $l_2$ (the number of type-II zones), and in the second place the desired temperatures $T_i$ ($i = l_1 + 1, ..., l$) enter into the zonal equations explicitly. The latter circumstance makes it possible to solve the system (5) by Newton's iteration method, which converges rapidly in a wide range.

The initial data for calculating mass transfer between the exchange zones as well as the coefficients of convective ex-

change are aerodynamic contours of the forward motion of the gases flows (sequence of passage through the zones) and the degree of fuel burnup [1]. This information makes it possible, in the first place, to evaluate the power of heat release due to gas combustion in the $i$th volume zone

$$Q_i^V = BQ_1^{\mathrm{w}}(\chi_{i+1} - \chi_i),$$

where $Q_i^V$ is the amount of heat released in the $i$th volume zone, kW; $B$ is the gas consumption for glassmaking, $\mathrm{m^3/sec}$; $Q_1^{\mathrm{w}}$ is the lowest working heat-producing capacity of the gas, $\mathrm{kJ/m^3}$; and, $\chi_i$ is the degree of fuel burnup in the $i$th volume zone, arb. units.

Taking into consideration the expression (2) for the convective heat flows $Q_i^{\mathrm{con}}$, the equation for calculating the mass flux $G_{\mathrm{p.c}}$ of the products of combustion has the form

$$G_{\mathrm{p.c}} = B(1 + \alpha L_0),$$

where $L_0$ is the stoichiometric air consumption for combustion (consumption factor $\alpha = 1.1$), $\mathrm{m^3/m^3}$.

Then, for all volume zones in the path of the gases except the first one ($i = n + 1, ..., l$), the total heat delivery due to gas motion will be

$$Q_i^{\mathrm{con}} = \sum_j \alpha_j F_j T_j - T_i \sum_j \alpha_j F_j + \sum_{j'} c_{j'}(G_{j'}^{\mathrm{g}} + G_{j'}^{\mathrm{a}} + G_{j'}^{\mathrm{p.c}})T_{j'} - 273\sum_{j'} c_{j'}(G_{j'}^{\mathrm{g}} + G_{j'}^{\mathrm{a}} + G_{j'}^{\mathrm{p.c}}) - c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}})T_i + 273c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}}) = \sum_k g_{ki}T_k - g_{ii}T_i + g_i^0,$$

where

$$g_i^0 = 273[(c_i G_i^{\mathrm{g}} - c_{j'} G_{j'}^{\mathrm{g}}) + (c_i G_i^{\mathrm{a}} - c_{j'} G_{j'}^{\mathrm{a}}) + (c_i G_i^{\mathrm{p.c}} - c_{j'} G_{j'}^{\mathrm{p.c}})];$$

$$g_{ki} = \begin{cases} 0 & \text{for } k \neq j, k \neq j', k \neq i; \\ \alpha_j F_j & \text{for } k = j; \\ c_{j'}(G_{j'}^{\mathrm{g}} + G_{j'}^{\mathrm{a}} + G_{j'}^{\mathrm{p.c}}) & \text{for } k = j'; \\ \sum_j \alpha_j F_j + c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}}) & \text{for } k = i. \end{cases}$$

For the first zone in the path of the gases

$$Q_i^{\mathrm{g}} = \sum_j \alpha_j F_j T_j - T_i \sum_j \alpha_j F_j + c_{\mathrm{g}} G_{\mathrm{g}} T_{\mathrm{g}} + c_{\mathrm{a}} G_{\mathrm{a}} T_{\mathrm{a}} + \sum_{j'} c_{j'} G_{j'}^{\mathrm{p.c}} T_{j'} - 273\left(c_{\mathrm{g}} G_{\mathrm{g}} + c_{\mathrm{a}} G_{\mathrm{a}} + \sum_{j'} c_{j'} G_{j'}^{\mathrm{p.c}}\right) - c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}})T_i + 273c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}}) = \sum_k g_{ki}T_k - g_{ii}T_i + g_i^0,$$

where

$$g_{ki} = \begin{cases} 0 & \text{for } k \neq j, k \neq j', k \neq i; \\ \alpha_j F_j & \text{for } k = j; \\ c_{j'} G_{j'}^{\mathrm{p.c}} & \text{for } k = j'; \\ \sum_j \alpha_j F_j + c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}}) & \text{for } k = i, \end{cases}$$

$$g_i^0 = 273[(c_i(G_i^{\mathrm{g}} + G_i^{\mathrm{a}} + G_i^{\mathrm{p.c}}) - c_{\mathrm{g}} G_{\mathrm{g}} - c_{\mathrm{a}} G_{\mathrm{a}} - \sum_{j'} c_{j'} G_{j'}^{\mathrm{p.c}}] + c_{\mathrm{g}} G_{\mathrm{g}} T_{\mathrm{g}} + c_{\mathrm{a}} G_{\mathrm{a}} T_{\mathrm{a}} + T_{\mathrm{p.c}} \sum_{j'} c_{j'} G_{j'}^{\mathrm{p.c}}.$$

For the surface zones

$$g_{ii} = \alpha_i F_i + K_i F_i = (\alpha_i + K_i) F_i;$$

$$g_i^0 = K_i F_i T_{\mathrm{ext}\,i},$$

where $g_{ii}$ is the coefficient of convective exchange, which takes account of heat exchange between the $i$th zone and the solid surface in contact with it; $K_i$ is the coefficient of heat transfer in the $i$th surface zone, $\mathrm{W/(m^2 \cdot K)}$; and, $T_{\mathrm{ext}\,i}$ is the temperature of the external surface of the masonry in the $i$th zone, K.

We shall now make use of the zonal equations (5) to construct a system of working equations of the model for surface zones of the molten glass and the lining as well as for the volume gas zones. Since the unknowns in the system of nonlinear equations of the resolvent zonal method are the temperatures of the zones, the equations corresponding to the type-I zones are eliminated from it.

For the surface zones of the molten glass with boundary conditions of the second kind, the nonlinear equation of heat transfer and heat balance assumes the form

$$\sum_k (A_{ki}T_k^4 + g_{ki}T_k) - g_{ii}T_i + A_{ii}T_i^4 - Q_{\mathrm{gm}} = 0 \quad \text{for } i = 1, ..., n_1, \tag{6}$$

where $A_{ii}$ is the coefficient of radiation exchange between the $i$th zone and the adjoining zones; $n_1$ is the number of type-II surface zones (of the molten glass); and, $Q_{\mathrm{gm}}$ is a source term, kW.

In turn,

$$Q_{\mathrm{gm}} = Q_{1.1} + Q_{1.2} + Q_{\mathrm{m.m}} - Q_{\mathrm{e}},$$

where $Q_{1.1}$ and $Q_{1.2}$ are the heat consumption for glassmaking and heating the molten glass, respectively, kW; $Q_{\mathrm{m.m}}$ represents the heat losses through the masonry of the melting tank, kW; and, $Q_{\mathrm{e}}$ is the AEH power, kW.

A unique relation between the resulting heat flow density and the internal temperature $T_i$ can be established for the surface zones of the masonry in the melting chamber. For this reason, the boundary conditions of the third kind are set by the coefficient of heat transfer $K_i$ and the temperature $T_{\mathrm{sur}}$ of the surrounding medium.

Under these conditions Eq. (5) assumes the form

$$\sum_k (A_{ki}T_k^4 + g_{ki}T_k) - g_{ii}T_i + A_{ii}T_i^4 -$$
$$K_i F_i (T_i - T_{\text{sur}}) + z_i = 0 \text{ for } i = n_1 + 1, ..., n, \quad (7)$$

where $z_i$ is the heat delivered (consumed) due to a type-I zone.

The heat-release power $Q_i^V$ due to fuel combustion is known for the volume gas zones. For this reason, for these zones the values of the resulting fluxes $\widetilde{Q}_i = -Q_i^V$ are given, and the temperatures $T_i$ are determined. Thus, the volume gas zones are type-II zones, for which

$$\sum_k (A_{ki}T_k^4 + g_{ki}T_k) - g_{ii}T_i + A_{ii}T_i^4 + g_i^0 + Q_i^V + z_i = 0$$
$$\text{for } i = n + 1, ..., m + n. \quad (8)$$

As a result, the numerical model of external heat exchange in the melting chamber of the gas-electric furnace is formalized by the closed system of zonal equations (6) – (8) for the temperatures $T_1, ..., T_i$.

When solving a system of nonlinear equations by Newton's method, the emissivity of the gas zone, the matrices of the proportionality coefficients, the matrices of the generalized proportionality coefficients, the matrices of the resolving generalized proportionality coefficients, as well as the coefficients of radiation and convection exchange are all calculated at each temperature iteration. The criterion of convergence of the iteration process is that two successive temperature fields in each zone coincide with error not exceeding $10^{-4}$.

The algorithm for calculating the coefficients of radiation exchange is presented in [2].

The numerical model developed by the present authors taken together with the mathematical model of the hydrodynamics of a melting tank [3] can be used to calculate the parameters of the thermal operation of high-capacity glassmaking furnaces with gas-electric heating.

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